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Abstract.--Theoretical diurnal variations in the meteor echo rate have been computed for the Harvard Radio Meteor Project radar system at Havana, Illinois. Meteor showers with radiant declinations $< +50$ degrees will be detected for about 3 hours per day while showers with radiant declinations $> +70$ degrees are continuously detected. The dependence of total echo rate on declination has been established, showing that the maximum numbers of meteors are detected from radiants at declinations of +70 to +75 degrees. The weighted distribution of observed meteors as a function of declination is symmetrical about zero declination.

The diurnal variation in the sporadic echo rate during March 1963 has been shown to be consistent with five concentrations of meteor radiants; one near the apex, two centers on the ecliptic and about 70 degrees from the apex, and two centers about 60 degrees north and south of the apex. A comparison of the observed and theoretical rates for March indicates that the average flux over the earth of sporadic meteors that produce trails with zenith line densities in excess of 4.5×10^{10} electrons/m is 40 km^{-2} hr^{-1}.

Introduction

The rate of meteor detection with any radio system depends on a number of factors involving the parameters of the radar equipment, the distribution of meteors in space, and the ablation of the meteors in the atmosphere. For a given radar system the response, in terms of the echo rate, to meteors from a given radiant is determined chiefly by the position of the echo plane with respect to the radiation pattern of the antennas. It has been shown (Elford, 1964a) that for an isotropic unit density distribution of radiants, the echo rate from radiants lying within an element of the celestial sphere of solid angle $d\Omega$ about the direction

(θ_r, φ_r) is given by

$$n(\theta_r, \varphi_r)\ d\Omega = A\ d\Omega \int_{-\pi/2}^{\pi/2} \left[G_T(\theta, \varphi) \cdot G_R(\theta, \varphi) \right]^{-\frac{c}{2}} f(\Phi)\ d\Phi \ , \quad (1)$$

where G_T and G_R are the power gains of the transmitting and receiving antennas; Φ is the azimuth angle of the reflection point measured in the echo plane; c is the exponent in the integral flux law; f is a slowly varying function of Φ ; and A is a constant involving the parameters of the radar equipment.

Let the density distribution of radiants over the celestial sphere with respect to an isotropic distribution be given by the function $g(\beta, \lambda-\lambda_\odot)$ where β is the celestial latitude and $\lambda-\lambda_\odot$ is the celestial longitude minus the longitude of the sun.

Then the total radio echo rate as a function of time is given by

$$n(t) = \iint_{\text{visible sky}} n(\theta_r, \varphi_r) \quad g(\beta, \lambda-\lambda_\odot)\ d\Omega \quad , \qquad (2)$$

where t is the local solar time on a given day of the year. The diurnal variation in the echo rate is thus found by numerical integration of equation (2) for each hour of the day.

Before proceeding to consider this general case, it is of interest to investigate the special case when $g(\beta, \lambda-\lambda_\odot)$ is defined for a small element of the celestial sphere and is zero elsewhere. This is equivalent to determining the diurnal variation in the echo rate for a meteor shower.

The Echo Rate for a Meteor Shower

Meteor radiants associated with a meteor shower lie within a small region of the celestial sphere. If all the meteors giving rise to radio echoes are considered to be associated with a meteor shower, then equation (2) for the echo rate as a function of time reduces to

$$n_{sh}(t) = \text{const.}\ n(\theta_r, \varphi_r) \ , \tag{3}$$

where the elevation and azimuth of the radiant are functions of the local solar time.

The theoretical echo rate of point radiants as a function of time has been computed for two types of observations carried out with the Harvard meteor radar system at Havana, Illinois: (a) single-station observations at site 3 using the single trough A for transmission and the single trough B for reception, and (b) 6-station observations using the requirements for orbit measurement, namely, that echoes must be obtained at a minimum of three sites, two of which must be sites 3 and 4. For the second type of observation the single trough A at site 3 was used for transmission, and signals were received at site 3 on the single trough B and at the other sites on Yagi antennas. The form of the diurnal variation in rate depends on the declination of the shower radiant, and the analysis has been carried out at intervals of declination of 5 degrees for northern declinations and intervals of 10 degrees for southern declinations. The value of the exponent c in the flux law was taken as -1.34. The results are presented as relative rate curves in Figures 1 and 2. Time zero refers to the time of culmination of the radiants.

The time of maximum rate occurs 2 to 6 hours after transit of the radiant, and for declinations < +50 degrees meteors are detected from a given radiant for about 3 hours per day. At higher declinations the period that the radiant is within the collecting area of the system increases rapidly and at declinations > +70 degrees the radiant is visible for the whole day. These curves can be used to weight the observed rates of showers in order to determine the relative flux of meteors associated with each shower.

In order to compare the strengths or average daily rates of showers whose radiants have different declinations, it is necessary to know the dependence of echo rate on declination. The number of meteors as a function of declination is readily found by integrating each diurnal rate curve over a period of 24 hours. The results are given in Figure 3 for the single and multi-station systems. To facilitate comparison of the curves, the maximum rates have been made equal. The duration of time that high declination radiants are within the collecting area of the system more than outweighs the weaker response of the system to these radiants, with the result that the maximum numbers of meteors are detected from radiants at declinations of +70 degrees to +75 degrees. The geometry of the 6-station system is such that there is an additional emphasis of medium declination radiants as evidenced by the bump in the curve at declination ~ +45 degrees.

The effect of a 30% variation in the flux law exponent on the distribution of meteors as a function of declination has been examined and the results are plotted in Figure 4. It is clear that the form of the distribution is relatively insensitive to the value of the exponent.

The Declination Distribution of Sporadic Meteors

It is clear that the observed number of meteors as a function of declination will be severely influenced by the emphasis of radiants of high declination. The true distribution can be found by weighting the observed data according to the theoretical curve for the multi-station system given in Figure 3. As an example, the distribution in declination of all sporadic meteors observed from January through August 1962 has been weighted for observational selection, and the observed and weighted distributions are shown in Figure 5. The observed distribution has a maximum at about declination +25 degrees, while the weighted distribution has its maximum at declination zero degree and shows a smooth fall-off toward +90 degrees. Although there are relatively few data for declinations < -10 degrees, the weighted histogram suggests that the true distribution with declination is symmetrical about zero declination.

For certain flux problems it is necessary to know the duration of time that a radiant of a given declination is within the collecting area of the radar system. This is shown in Figure 6 for the conditions for orbit determination. The limiting rates have been taken as 1% and 4% of the peak rate.

The Diurnal Echo Rate for a Given Distribution of Radio Meteor Radiants Over the Celestial Sphere

For the general case where the meteor radiants are distributed over the celestial sphere, the total echo rate at any time is given by the numerical integration of equation (2). This integration can be carried out either in terms of the celestial coordinates $(\beta, \lambda-\lambda_{\odot})$, or in terms of the altitude-azimuth coordinates (θ_r, φ_r). Since the value of the response function $n(\theta_r, \varphi_r)$ is zero over much of the range of altitude and azimuth, it is more efficient to carry out the integration in this coordinate system. We then must know β and $\lambda - \lambda_{\odot}$ as functions of θ_r and φ_r for any given day of the year. The relationships are obtained by five successive transformations of axes. These are given in the Appendix.

The theoretical echo rate of sporadic meteors has been computed for the meteor radar system at Havana, Illinois, using the sporadic radiant density distribution given in Figure 7.

This distribution was obtained from measurements of meteor orbits made with the same system during the period January - August 1962. The manner in which this distribution is derived from the orbital data is described elsewhere (Elford et al., 1964b). It is assumed that the distribution of sporadic radiants is symmetrical about the ecliptic. This distribution is characterized by five concentrations of activity; a center of activity near the apex, two centers of activity on the ecliptic and about 70 degrees in longitude from the apex, and two centers of activity 55 degrees north and south of the apex.

Diurnal echo rates were calculated for the equinoxes and the solstices and the results are given in Figure 8. In general, each curve shows three peaks of activity. The peak near 0930 is due to the concentration of radiants centered on longitude 270 degrees, and the other two peaks are due to the concentrations centered on the positions (0°, 200°) and (0°, 340°). At the time of the autumnal equinox, the high-latitude concentration remains within the collecting area of the antennas for the whole day. As a consequence, the theoretical daily echo rate during September is more than twice as large as the theoretical daily rate during March. It should be emphasized that these four diurnal rate curves have been calculated on the assumption that the form of the distribution of sporadic radiants is essentially the same from month to month, and that the total number of meteors incident on the earth per day is constant throughout the year. A comparison of these curves with the observed diurnal rates would readily check this assumption. On the other hand, any severe discrepancies between the observed and theoretical rate curve for any month could be removed by modifying the radiant density distribution.

Total Meteor Impact Rate on the Earth

Let the incident flux of meteors whose radiants lie within an element of the celestial sphere of solid angle $d\Omega$ about the direction (θ, φ) and which produce trails with zenith line densities greater than q_z be represented by

$$N_1(q_z) \; g(\theta, \varphi) \; d\Omega \;\; .$$

This is the flux across a plane normal to the meteor paths. The function $g(\theta, \varphi)$ is the distribution of sporadic meteors over the celestial sphere relative to an isotropic distribution. The function $N_1(q_z)$ is the flux of meteors per unit solid angle for a unit density distribution of radiants producing zenith line densities greater than q_z. It is assumed that the distribution function N_1 can be approximated by the power law

$$N_1(q_z) = K \; q_z^c \;\; .$$

The value of the exponent c which has been discussed previously (Elford, 1964a), lies between -1.0 and -1.3 for sporadic meteors.

The number of sporadic meteors per unit time from an element $d\Omega$ of the celestial sphere in the direction (θ, φ) that cross an element of area dA of a horizontal plane whose normal OZ makes an angle χ with the radiant direction is given by

$$N_1(q_z)\ g(\theta, \varphi)\ d\Omega\ dA \cos\chi \quad .$$

Then the total number of meteors dn incident per unit time on the whole earth from the element $d\Omega$ of the celestial sphere is found by integrating over the surface of a hemisphere symmetrical about OZ,

$$\text{i.e., } dn = \int_0^{2\pi} \int_0^{\pi/2} N_1\ g(\theta, \varphi)\ d\Omega\ R_E^2 \sin\chi \cos\chi\ d\chi\ d\Phi$$

$$= N_1\ g(\theta, \varphi)\ d\Omega\ \pi R_E^2 \quad .$$

The total rate of incidence of meteors on the whole earth from all sporadic radiants is found by integrating over the celestial sphere,

$$\text{i.e., } n = \pi R_E^2\ N_1 \iint g(\theta, \varphi)\ d\Omega$$

$$= \pi N_1\ (4\pi R_E^2) \quad ,$$

since $\iint g(\theta, \varphi) = 4\pi$ by definition.

Thus the average number of meteors incident per unit time on a unit area of the earth's surface is πN_1 .

An estimate of the average number of sporadic meteors incident on the earth per hour that produce trails with line densities in excess of 3.3×10^{10} electrons/m was made from rate measurements taken in March 1963. The observed diurnal echo rate is given in Figure 9. This curve is a composite of four sets of observations made at site 3 on March 12, 13, 14, and 15. A double-trough antenna was used for transmission and a single-trough antenna for reception. Wherever the curves overlap there is excellent agreement in the rates. A theoretical echo rate was calculated using the known response function for the system and the radiant density distribution obtained from the orbit observations of March 1962 (Elford et al., 1964b). The radiant distribution for March is similar to the January - August average given in Figure 8, but the concentration north of the apex was centered on latitude 60 degrees and was approximately 50% more dense. The theoretical echo rate is shown by the dashed line in Figure 9. The scale for the theoretical curve has been adjusted to give the best match between the theoretical and observed rates. Since the theoretical rates are based on data obtained during 1962, and the observed rates refer to 1963, the agreement between the curves is very satisfactory. The agreement could have been improved by modifying the radiant density distribution, but it was considered that further adjustments would be marginal and not warranted by the accuracy of the data. The sharp peak in the observed rates at 0915 is attributed to several minor streams which were active on March 12.

The theoretical rate was calculated for the following conditions,

$$P_T = 1.5 \times 10^6 \text{ watts}$$

$$P_R = 1.0 \times 10^{-12} \text{ watts}$$

$$K = 3.6 \times 10^{10} \text{ m}^{-2} \text{ hr}^{-1}$$

$$C = -1.343 \ .$$

The actual operating conditions were

$$P_T = 1.2 \times 10^6 \text{ watts}$$

$$P_R = 2.2 \times 10^{-13} \text{ watts} .$$

The choice of the value of K in the theoretical calculations was quite arbitrary. The actual value is found from a comparison of the observed and theoretical rates. Let the subscript 1 refer to the theoretical rates and the subscript 2 to the observed rates. It was shown in a previous report (Elford, 1964a) that the echo rate n is related to the parameters given above by an expression of the form

$$n \propto K \left(\frac{P_R}{P_T} \right)^{c/2} .$$

Then

$$K_2 = K_1 \frac{n_2}{n_1} \left(\frac{P_{R_1}}{P_{R_2}} \cdot \frac{P_{T_2}}{P_{T_1}} \right)^{c/2} .$$

From a comparison of the curves in Figure 9 the ratio $n_2/n_1 = 0.16$. Whence

$$K_2 = 2.4 \times 10^9 \text{ m}^{-2} \text{ hr}^{-1} .$$

The minimum detectable line density was determined for the following conditions:

$$P_T = 1.2 \times 10^6 \text{ watts}$$

$$P_R = 2.2 \times 10^{-13} \text{ watts}$$

$$G_T = 175$$

$$G_R = 105$$

$$R = 1.3 \times 10^5 \text{ meters}$$

$$\lambda = 7.3 \text{ meters} \quad .$$

The resultant value for q_D was 3.3×10^{10} electrons/m.

If we assume a mean radiant elevation of 45 degrees, the limiting zenith line density is $\sim 4.5 \times 10^{10}$ electrons/m. Hence the value of N_1, the flux of meteors that produce zenith line densities greater than 4.5×10^{10} electrons/m, is 1.2×10^{-5} m^{-2} hr^{-1}.

Thus the average rate of sporadic meteors incident on the earth during March that produce trails with zenith line densities in excess of 4.5×10^{10} electrons/m is ~ 40 km^{-2} hr^{-1}.

APPENDIX

Conversion of Altitude-Azimuth Coordinates (θ, φ) to Celestial Coordinates $(\beta, \lambda-\lambda_{\odot})$.

Let θ_0 and φ_0 be the elevation and azimuth of a point on the celestial sphere where the azimuth is measured from north through east. Let β be the celestial latitude and $\lambda-\lambda_{\odot}$ the celestial longitude minus the longitude of the sun. We need to know β and $\lambda-\lambda_{\odot}$ as functions of θ_0 and φ_0. The relationships are obtained by five transformations as follows.

(1) Convert from azimuth measured from north through east to azimuth measured from south through east. Let the new coordinates be (θ_1, φ_1). Then

$$\theta_1 = \theta_0$$

$$\varphi_1 = \pi - \varphi_0 \quad .$$

And the new direction cosines are

$$\ell_1 = \cos\varphi_1 \cos\theta_1 = -\cos\varphi_0 \cos\theta_0$$

$$m_1 = \sin\varphi_1 \cos\theta_1 = \sin\varphi_0 \cos\theta_0$$

$$n_1 = \sin\theta_1 = \sin\theta_0 \quad .$$

(2) Rotate about the east-west axis (Oy_1) by $-(90 - L)$ where L is the latitude of the station. The new coordinates are negative hour angle $(-h)$ and declination (δ). The transformation is

$$(\ell_2, m_2, n_2) = (\ell_1, m_1, n_1)\, T_2 \quad .$$

where T_2 =

$$\begin{pmatrix} \sin L & 0 & -\cos L \\ 0 & 1 & 0 \\ \cos L & 0 & \sin L \end{pmatrix} .$$

(3) Add the sidereal time (t) to the negative hour angle to get the right ascension (α). This transformation is achieved by a rotation of -t about Oz_2,

$$\text{i.e., } T_3 = \begin{pmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

(4) Rotate about the vernal equinox (which is the axis $\alpha = 0$ $\delta = 0$, i.e., Ox_3, by the obliquity (ϵ). The new coordinates are celestial longitude (λ) and celestial latitude (β). This transformation is achieved by a totation of $+\epsilon$ about Ox_3,

$$\text{i.e., } T_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & -\sin\epsilon \\ 0 & \sin\epsilon & \cos\epsilon \end{pmatrix} .$$

Let the direction cosines in this system be (ℓ_4, m_4, n_4) .

(5) Subtract the sun's longitude ($\lambda_\odot$) from the celestial longitude λ .

For computational purposes it is useful to write out the direction cosines (ℓ_4, m_4, n_4) in terms of (ℓ_1, m_1, n_1).

Let the product of the three transformations T_2, T_3, and T_4 be T .

Then $T = [T_2]\ [T_3]\ [T_4]$

$$= \begin{bmatrix} (SL) \cos t & (SL.CE) \sin t - (CL.SE) & -(SL.SE) \sin t - (CL.CE) \\ - \sin t & (CE) \cos t & -(SE) \cos t \\ (CL) \cos t & (CL.CE) \sin t + (SL.SE) & -(CL.SE) \sin t + (SL.CE) \end{bmatrix} ,$$

where

$$SL = \sin L \quad ; \quad CL = \cos L$$

$$SE = \sin \epsilon \quad ; \quad CE = \cos \epsilon$$

and

$$(\ell_4, m_4, n_4) = (\ell_1, m_1, n_1)\ T \quad .$$

Hence,

$$\ell_4 = \ell_1[(SL) \cos t] - m_1[\sin t] + n_1[(CL) \cos t],$$

$$m_4 = \ell_1[(SL.CE) \sin t - (CL.SE)] + m_1[(CE) \cos t] + n_1[(CL.CE) \sin t + (SL.SE)] \ ,$$

$$n_4 = -\ell_1[(SL.SE) \sin t + (CL.CE)] - m_1[(SE) \cos t] - n_1[(CL.SE) \sin t - (SL.CE)] \ ,$$

whence

$$\theta_4 = \text{arc sin } n_4 \ ,$$

$$\varphi_4 = \text{arc tan } \frac{m_4}{\ell_4} \ ,$$

and

$$\lambda - \lambda_\odot = \varphi_4 - \lambda_\odot \ ,$$

$$\beta = \theta_4 \quad .$$

For any given day of the year, the longitude of the sun and the sidereal time for zero hours U.T. can be found from the Almanac.

REFERENCES

Elford, W. G. 1964a. Calculation of the response function of the Harvard radio meteor project radar system. Harvard Radio Meteor Project Research Report No. 8, October.

Elford, W. G. et al., 1964b. Harvard Radio Meteor Project Research Report No. 10 (to be published).

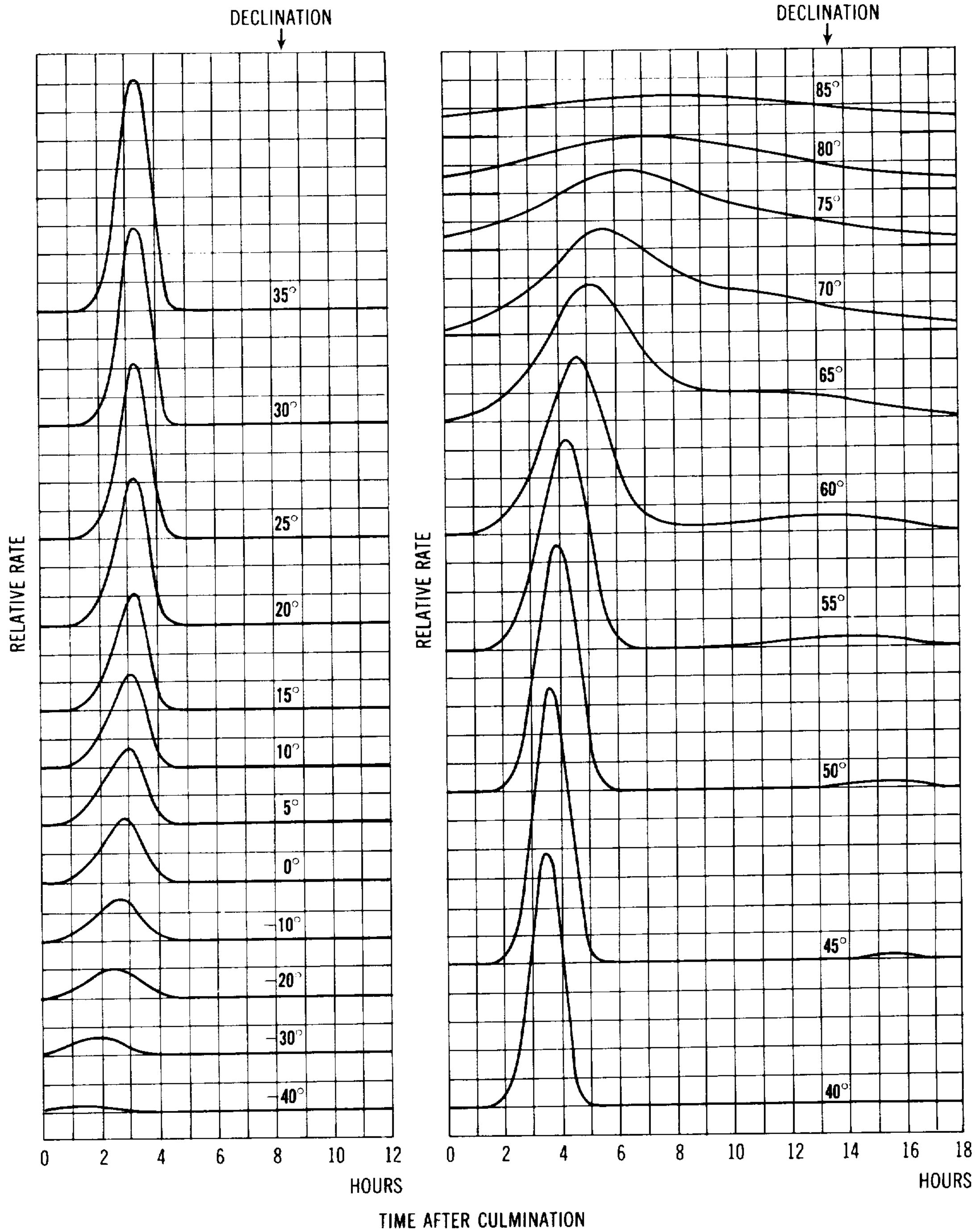

Figure 1.--Theoretical diurnal variation of the single-station radar echo rate for a point radiant. Calculations carried out for observations made at site 3; transmission on single trough A, reception on single trough B. Flux law exponent c = -1.34.

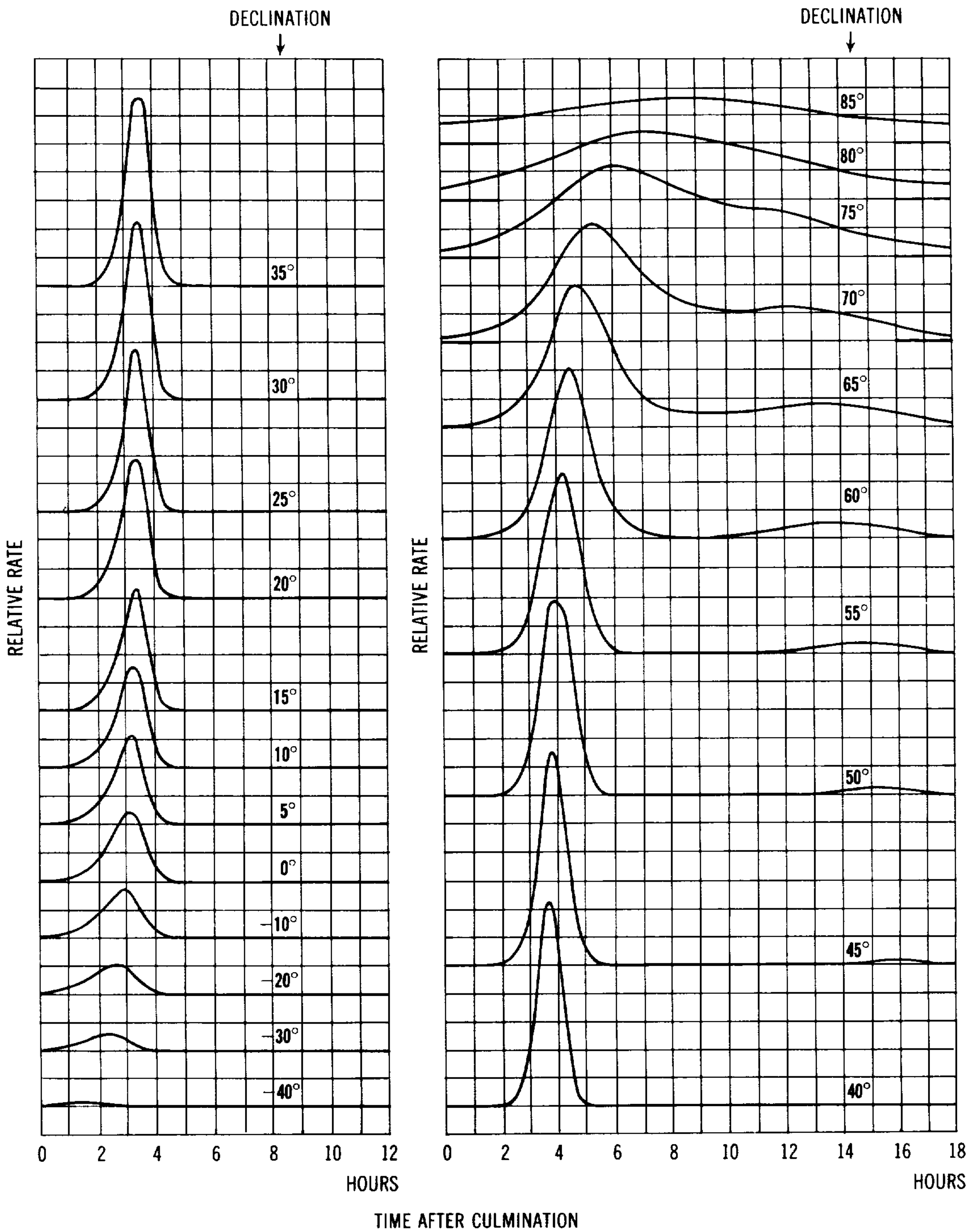

Figure 2.--Theoretical diurnal variation of the multi-station radar echo rate for a point radiant. Calculations carried out for the conditions that echoes must be obtained at a minimum of three sites, two of which must be sites 3 and 4. Transmission on single trough A at site 3, reception on single trough B at site 3 and on Yagi antennas at sites 1, 2, 4, and 6.

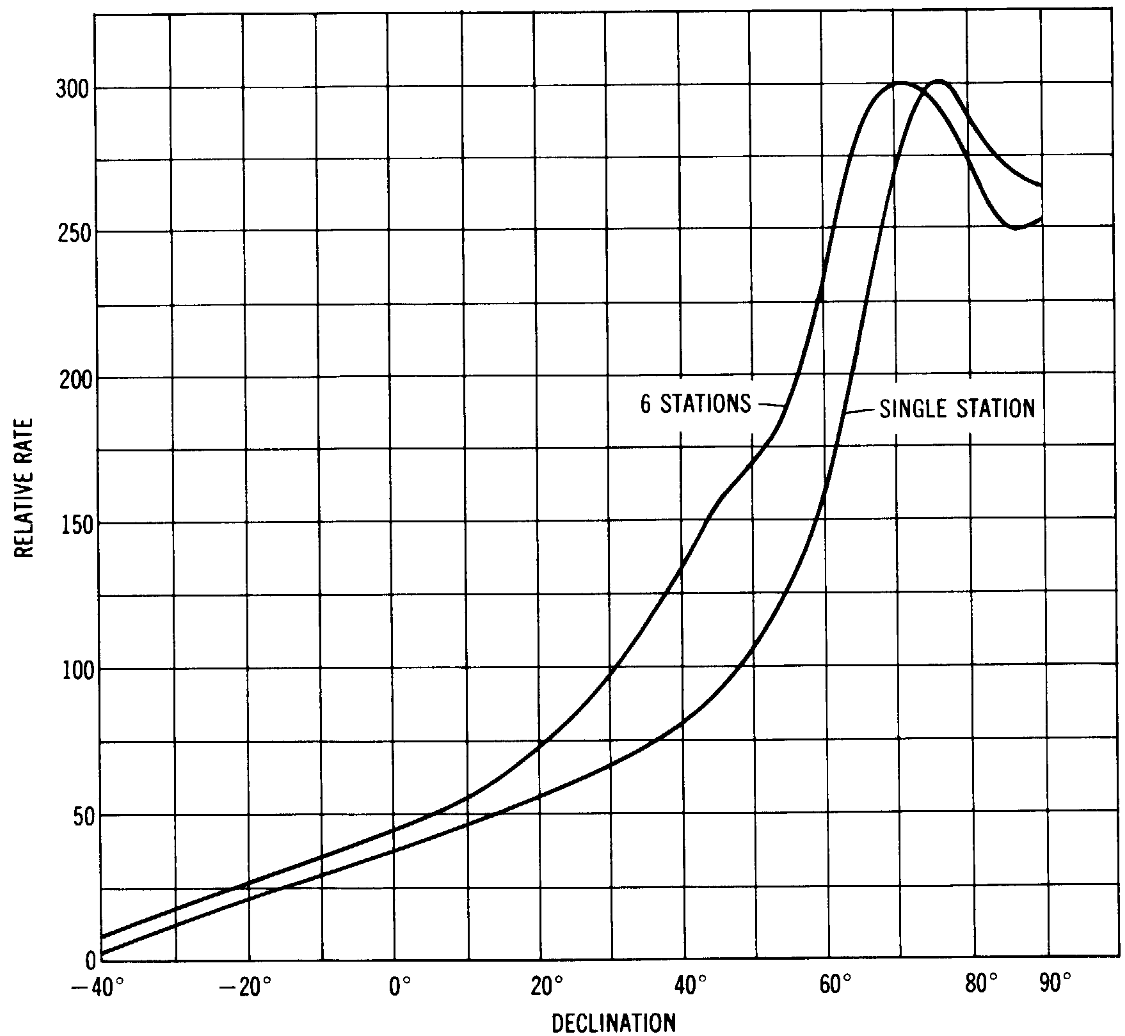

Figure 3.--Radar echo rate as a function of declination for the single and multi-station systems. The curves have been normalized to the same maximum echo rate.

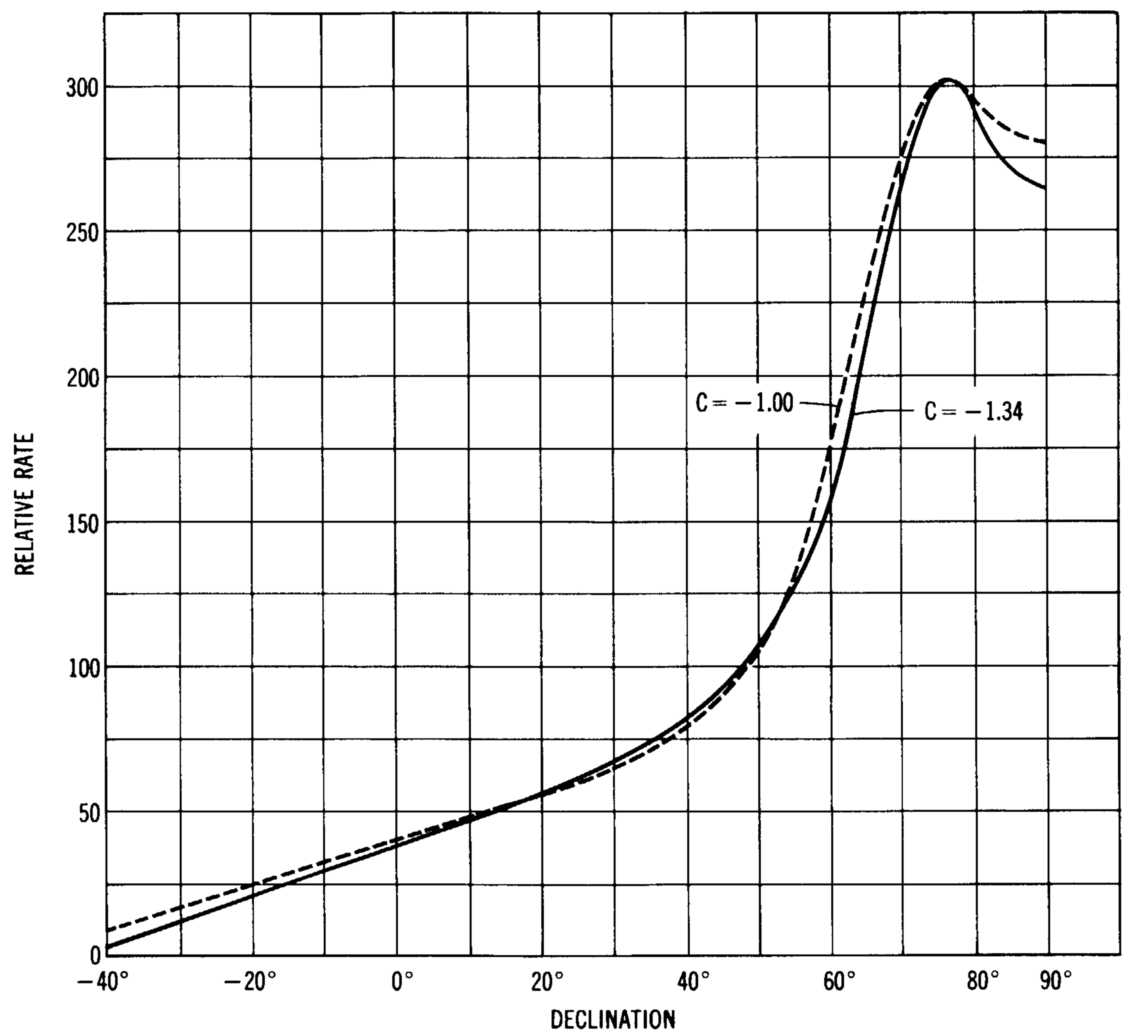

Figure 4.--Radar echo rate as a function of declination for two values of the exponent c in the flux law. The curves have been normalized to the same maximum echo rate.

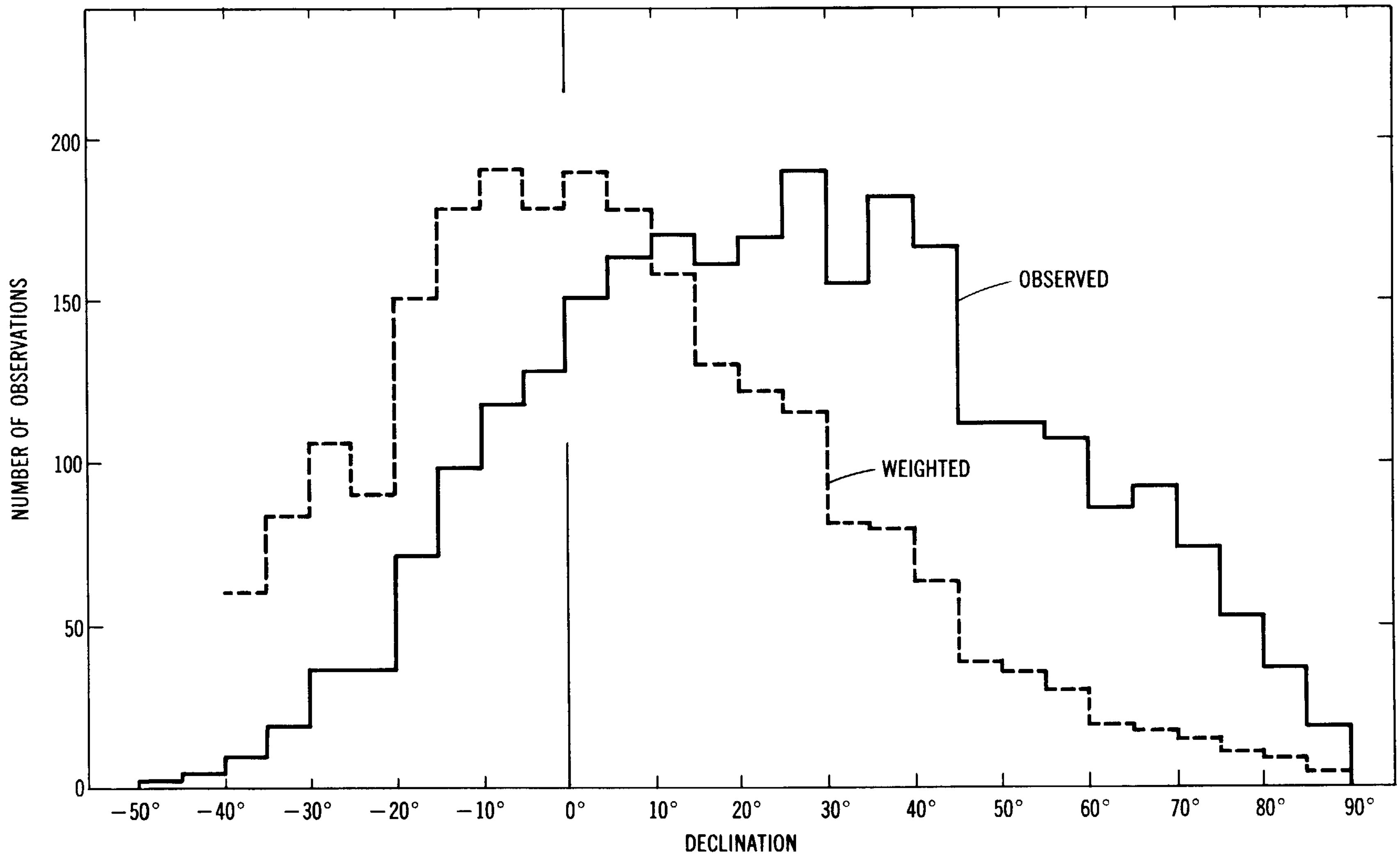

Figure 5.--Observed and weighted distributions of sporadic meteors as a function of declination for the period January - August 1962.

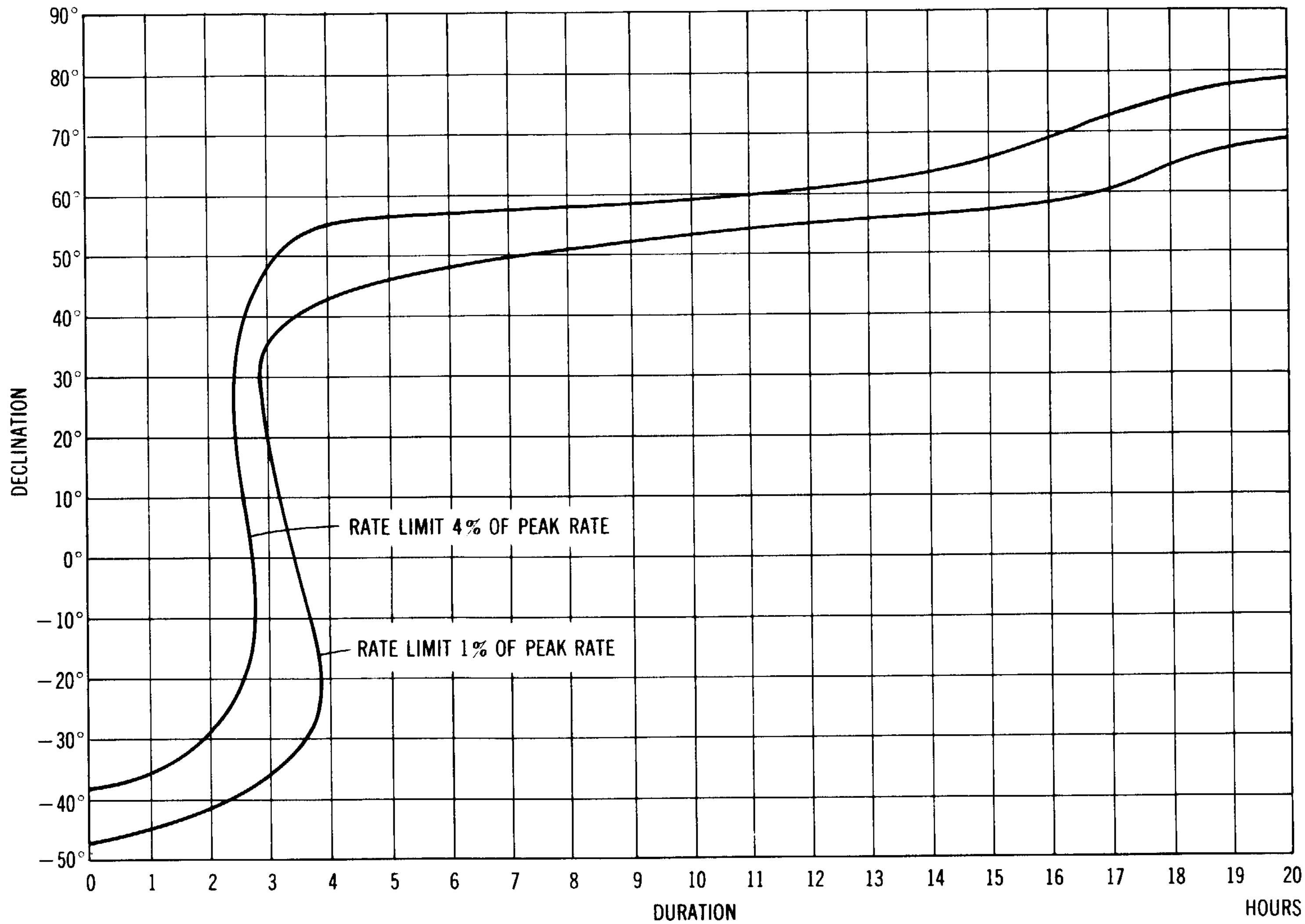

Figure 6.--Duration of time that a radiant of a given declination is within the collecting area of the 6-station system.

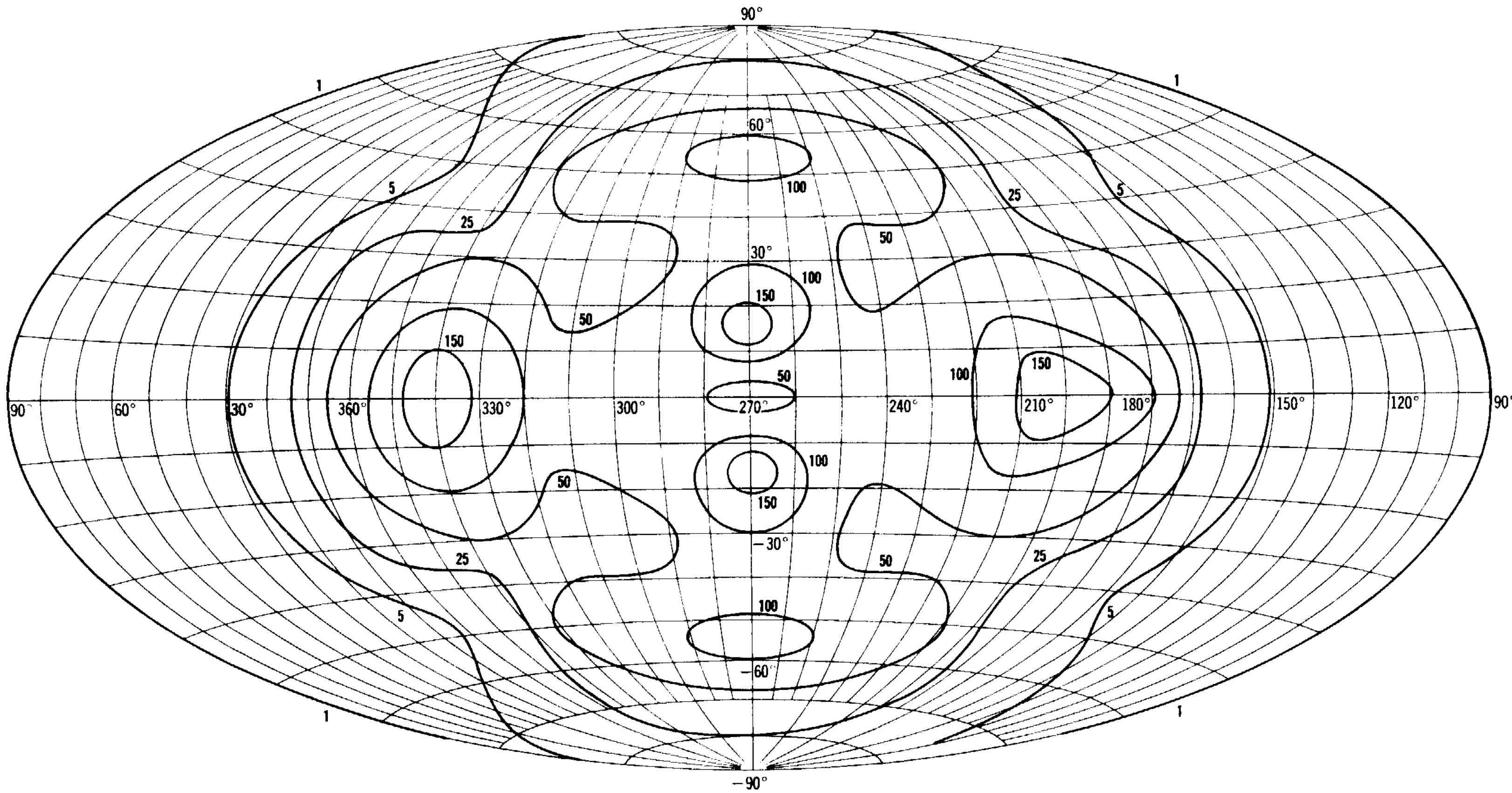

Figure 7.--The average density distribution of sporadic radiants for the period January - August 1962.

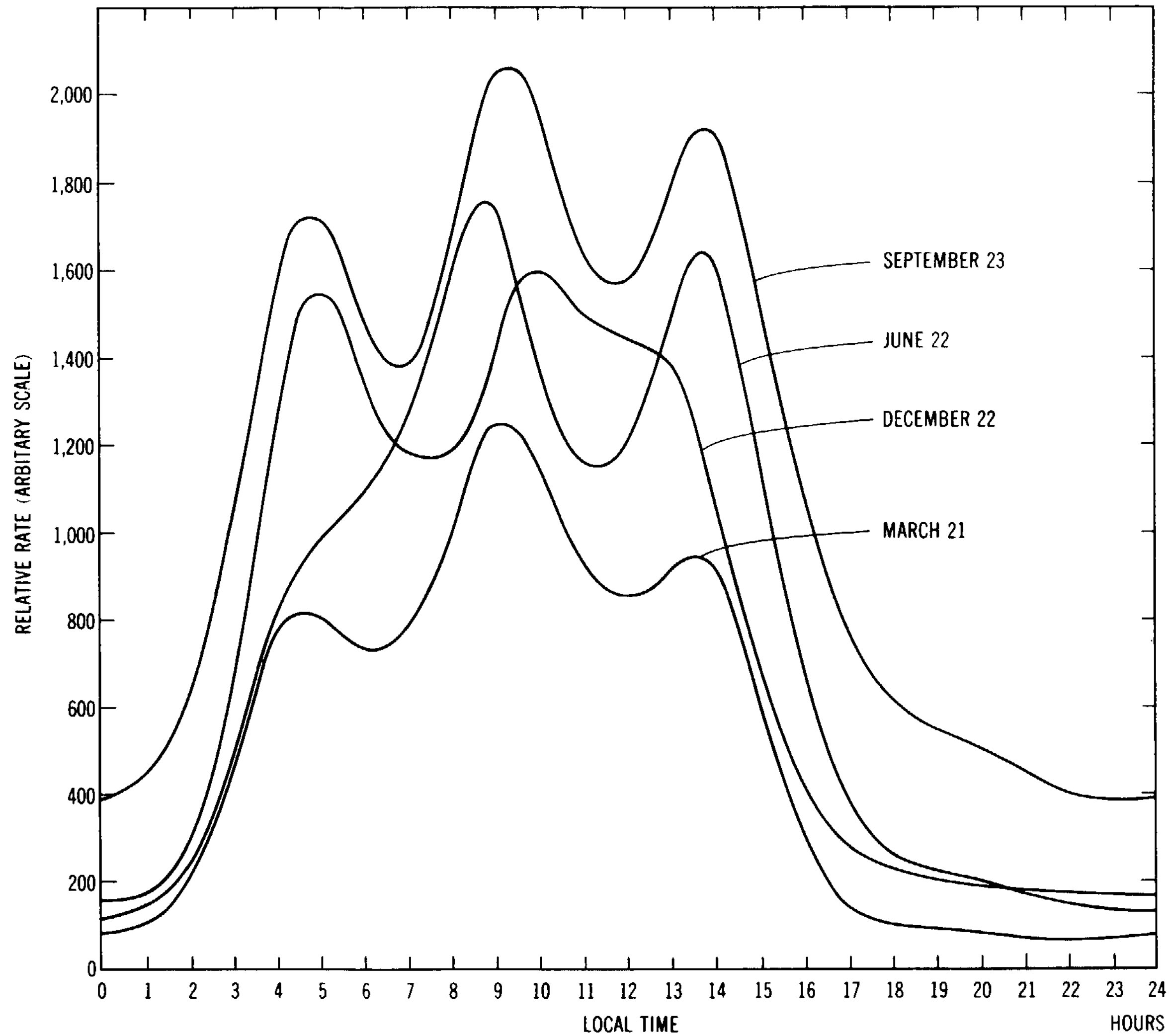

Figure 8.--Theoretical diurnal variation in the echo rates at the equinoxes and solstices based on the sporadic radiant distribution given in Figure 7.

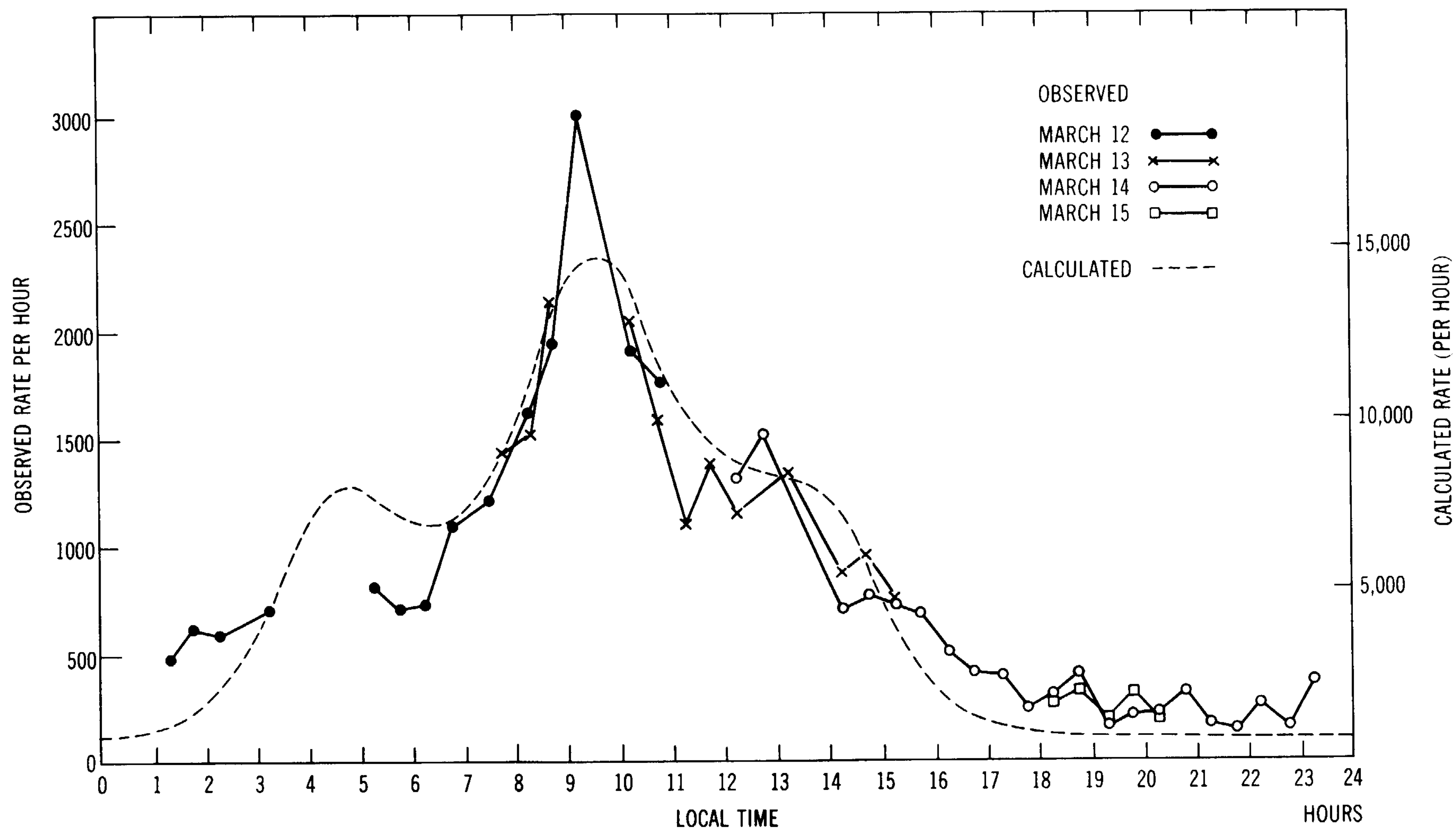

Figure 9.--Observed and theoretical diurnal variation in echo rates for March 1963.